

***CP* Asymmetries in Radiative B Decays with R-parity Violation**Eung Jin Chun^{a,b}, Kyuwan Hwang^c and Jae Sik Lee^b^a*Department of Physics, Lancaster University, Lancaster LA1 4YB, UK*^b*Korea Institute for Advanced Study, Seoul 130-012, Korea*^c*Department of Physics, KAIST, Taejon 305-701, Korea*

Abstract

We analyze the effect of R-parity violation in the minimal supersymmetric standard model on the *CP* asymmetries in $b \rightarrow s\gamma$ decay. The direct and mixing-induced *CP* asymmetries arising from the lepton number violating couplings are strongly constrained by the current experimental limits on the corresponding couplings. Allowing a heavy neutrino ($m_{\nu_\tau} \sim 10$ keV) and a moderate mass splitting of sfermions, the direct *CP* asymmetry around 15 % and the nearly maximal mixing-induced *CP* asymmetry ($\sim 100\%$) can be realized, depending on the R-parity conserving contributions to the radiative b decay. With the baryon number violating couplings, only the mixing-induced *CP* asymmetry arises and it can be maximal provided there is a the similar sfermion mass splitting.

PACS number(s):11.30.Er,12.60.Jv,13.20.He,13.40.Hq

I. INTRODUCTION

CP violation in radiative B decays may provide a promising tool for probing new physics beyond the Standard Model (SM). The direct CP asymmetry in the radiative $b \rightarrow s\gamma$ decay defined by [1]

$$A_{CP}^{b \rightarrow s\gamma}(\delta) = \frac{\Gamma(\overline{B} \rightarrow X_s \gamma) - \Gamma(B \rightarrow X_{\overline{s}} \gamma)}{\Gamma(\overline{B} \rightarrow X_s \gamma) + \Gamma(B \rightarrow X_{\overline{s}} \gamma)} \Big|_{E_\gamma > (1-\delta)E_\gamma^{\max}} \quad (1)$$

is below 1 % in the SM [2]. Therefore, the observation of a sizable CP asymmetry would be a clean signal of new physics and may further discriminate various extensions of the SM. Recent model-independent analyses of CP violating effects in the inclusive decays $B \rightarrow X_s \gamma$ in terms of the Wilson coefficients of the dipole moments operators have shown that models with enhanced chromo-magnetic dipole moments can naturally provide a large asymmetry [1] and it can reach up to 30 % accommodating the observed branching ratios of $b \rightarrow s\gamma$ and $b \rightarrow sg$ decays [3]. The specific predictions for the CP asymmetry are of course model-dependent. The left-right model can yield A_{CP} at the level of 1 % [4], and a two Higgs doublet model up to 10 % [5]. In supersymmetric extensions of the Standard Model, there exists additional CP phases which would give rise to large CP violating effects. In the context of minimal supergravity models [6], the direct CP asymmetry turns out to be less than 2 % due to strong constraints on supersymmetric CP violating phases from the neutron and electron electric dipole moments [7]. The asymmetry can be enlarged in non-minimal models up to 7 % with more freedom in CP phases [8], or up to 15 % with generic sfermion mixing [9,3].

Another important CP violating observable is the mixing-induced CP asymmetry in exclusive radiative B decays [10] which can occur for radiative $B_q \rightarrow M_q \gamma$ decays, where $M_{q=d,s}$ is any hadronic self-conjugate state with CP eigenvalue $\xi = \pm 1$. The CP asymmetry in the time-dependent decay rates $\Gamma(t)$ for $B_q \rightarrow M_q \gamma$ and $\bar{\Gamma}(t)$ for $\bar{B}_q \rightarrow M_q \gamma$ is then

$$A(t) \equiv \frac{\Gamma(t) - \bar{\Gamma}(t)}{\Gamma(t) + \bar{\Gamma}(t)} = \xi A_M \sin(\phi_M - \phi_L - \phi_R) \sin(\Delta m t), \quad A_M \equiv \frac{2|C_{7L}C_{7R}|}{|C_{7L}|^2 + |C_{7R}|^2} \quad (2)$$

where ϕ_M and Δm are the phase and the mass difference of B_q - \bar{B}_q mixing, respectively, and $C_{7L,7R}$ are the effective coefficients of the left-handed and right-handed dipole moment operators for the $b \rightarrow q\gamma$ decays, and $\phi_{L,R}$ are their phases, respectively. In the standard model, such asymmetries are expected to be small as one has $C_{7R}/C_{7L} \approx m_q/m_b$ and thus A_M of order 1 % and 10 % for the $b \rightarrow d\gamma$ and $b \rightarrow s\gamma$ decays, respectively. Unlike the direct CP asymmetry, the left-right symmetric model may allow the mixing-induced asymmetry up to 50 % [10]. The supersymmetric model with generic sfermion mixing can yield even larger asymmetry up to 90 % [9].

In this paper, we will analyze the effects of R-parity violation in the Minimal Supersymmetric Standard Model (MSSM) on the CP asymmetries in radiative B decays. R-parity violation in the MSSM introduces a large number of trilinear couplings which violate lepton and baryon number. These additional couplings can surely be sources of flavor and CP violation, which might lead to a huge effect on the CP -odd observables in the B decays.

CP violating effects of R-parity violation in the hadronic B decays have been considered previously in Refs. [11–15], showing that significant modifications to the SM predictions can follow from R-parity violation. This work is devoted to the analysis of the CP asymmetries in the radiative B decay. As we will see, contrary to the cases with hadronic B decays, various experimental constraints on the R-parity violating couplings coming particularly from rare B decays strongly limit the amount of the direct CP asymmetry in the $b \rightarrow s\gamma$ decay whereas a large mixing-induced CP asymmetry can be induced from, in particular, baryon number violation.

This paper is organized as follows. In section II, we analyze the general effective Hamiltonian describing the $b \rightarrow s\gamma$ decay including the new operators induced by R-parity violation. We derive the anomalous dimensions and evolution matrix with the enlarged operator set. In section III, we discuss the CP asymmetries arising from lepton number violating couplings with which we can have both the direct and mixing-induced CP asymmetry. We deal with both cases in separated subsections. In section IV, we discuss the CP asymmetries arising from baryon number violating couplings, in which case only mixing-induced CP asymmetry can be obtained. We conclude in section V.

II. R-PARITY VIOLATION AND EFFECTIVE HAMILTONIAN

Let us begin our discussion with defining our choice of the basis to describe the R-parity violating couplings. For comparison with experiments, it is convenient to work with R-parity violating couplings defined in the quark and lepton mass eigenbasis. In this prescription, we leave the neutrinos in the charged lepton mass eigenbasis since neutrinos can be taken to be massless for our purpose. The full superpotential of the MSSM fields including generic R-parity violating couplings is then given by

$$W = \mu H_1 H_2 + h_i^e H_1 L_i E_i^c + h_i^d (H_1^0 D_i D_i^c - H_1^- V_{ij}^\dagger U_j D_i^c) + h_i^u (H_2^0 U_i U_i^c - H_2^+ V_{ij} D_j U_i^c) \\ + \lambda_{ijk} (L_i^0 E_j - L_j^0 E_i) E_k^c + \lambda'_{ijk} (L_i^0 D_j D_k^c - E_i V_{jl}^\dagger U_l D_k^c) + \frac{1}{2} \lambda''_{ijk} U_i^c D_j^c D_k^c, \quad (3)$$

where $L_i = (L_i^0, E_i)$ and $Q_i = (U_i, D_i)$ are the lepton and quark $SU(2)$ doublets, and E_i^c, U_i^c, D_i^c are the $SU(2)$ singlet anti-lepton and anti-quark superfields. Here V_{ij} is the Cabibbo–Kobayashi–Maskawa (CKM) matrix of quark fields. In order to ensure the longevity of proton, the products $\lambda'\lambda''$ have to be highly suppressed [16]. For this reason, one usually assumes lepton or baryon number conservation to discard the couplings λ/λ' or λ'' , respectively. In this paper, we discuss both cases separately: one with R-parity and lepton number violation with nonvanishing λ' , and the other with R-parity and baryon number violation with λ'' . The couplings λ will be irrelevant for our discussion.

The presence of R-parity violating couplings in Eq. (3) give rise to new contributions to the radiative decay $b \rightarrow s\gamma$ through one-loop diagrams exchanging sleptons or squarks [17] as depicted in FIG. 1. Note that R-parity violation may induce equally sizable dipole moments of the left-handed and right-handed type, respectively labeled by L and R as follows:

$$O_{7L} \propto \overline{s_L^\alpha} \sigma^{\mu\nu} b_R^\alpha F_{\mu\nu}, \quad O_{7R} \propto \overline{s_R^\alpha} \sigma^{\mu\nu} b_L^\alpha F_{\mu\nu}. \quad (4)$$

In the SM and many extensions of it, the coefficients of the second one is usually suppressed by the factor m_s/m_b . In our case with lepton or baryon number violation in the MSSM, the operator O_{7L} and O_{7R} are generated for nonvanishing combinations of couplings;

$$\lambda'_{n3j}\lambda'^*_{n2j} \quad \text{and} \quad \lambda'_{nj2}\lambda'^*_{nj3}/\lambda''_{n12}\lambda''^*_{n23}, \quad (5)$$

respectively. With the couplings in Eq. (5), there arise also other four-quark operators which should be taken into account in the complete effective Hamiltonian describing the $b \rightarrow s\gamma$ decay. The whole set of the effective operators arising from R-parity violation can be described by a simple generalization of the SM operator space by separating the standard operators $O_{3,4,5,6}$ for each quark flavor $q = u, c, d, s, b$. That is, we introduce the additional operators,

$$\begin{aligned} O_{3L}^q &= \overline{s_L^\alpha} \gamma^\mu b_L^\alpha \overline{q_L^\beta} \gamma_\mu q_L^\beta, & O_{4L}^q &= \overline{s_L^\beta} \gamma^\mu b_L^\alpha \overline{q_L^\alpha} \gamma_\mu q_L^\beta, \\ O_{5L}^q &= \overline{s_L^\alpha} \gamma^\mu b_L^\alpha \overline{q_R^\beta} \gamma_\mu q_R^\beta, & O_{6L}^q &= \overline{s_L^\beta} \gamma^\mu b_L^\alpha \overline{q_R^\alpha} \gamma_\mu q_R^\beta, \end{aligned} \quad (6)$$

where α, β are the color indices. The dipole moment operators for the $b \rightarrow s\gamma$ and $b \rightarrow sg$ are defined by

$$\begin{aligned} O_{7L} &= \frac{e}{16\pi^2} m_b \overline{s_L^\alpha} \sigma^{\mu\nu} b_R^\alpha F_{\mu\nu}, \\ O_{8L} &= \frac{g_s}{16\pi^2} m_b \overline{s_R^\alpha} \sigma^{\mu\nu} T_{\alpha\beta}^a b_L^\beta G_{\mu\nu}^a. \end{aligned} \quad (7)$$

We also have the right-handed counterpart of the operators which can be obtained by the exchange $L \leftrightarrow R$ in Eqs. (6) and (7).

The effective Hamiltonian at scale $\mu \leq \mathcal{O}(m_W)$ relevant for the $b \rightarrow s\gamma$ decay is now described in terms of the enlarged set of operators as follows:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \lambda_t \left[\sum_{i=1}^8 C_{iL}(\mu) O_{iL}(\mu) + \sum_q \sum_{j=3}^6 C_{jL}^q(\mu) O_{jL}^q(\mu) + (L \leftrightarrow R) \right], \quad (8)$$

where $\lambda_t = V_{ts}^* V_{tb}$, G_F is the Fermi constant and C 's are the Wilson coefficients which will be determined later. The operators O_{iL} with $i = 1, \dots, 8$ are those considered in the SM [18]. Note that $O_{1L,2L} = O_{3L,4L}^c$ and $O_{iL} = \sum_q O_{iL}^q$ for $i = 3, \dots, 6$.

Given the Wilson coefficients C_i, C_j^q including the contributions from R-parity violation at the weak scale $\mu = m_W$, we need to calculate those at the scale $\mu \sim m_b$ through the renormalization group (RG) evolution. Since the QCD running do not mix the left-handed and right-handed set of operators and its effect is identical, it is enough to calculate the RG equation at the one sector. At the leading order, it is rather straightforward to calculate the anomalous dimension matrix in the extended operator basis following the standard calculation [18]. At this point, let us recall that it is convenient to use the so-called ‘‘effective coefficients’’ [19] which are free from the regularization-scheme dependency in the mixing between the sets $O_i^{(q)}$ with $i = 1, \dots, 6$ and $O_{7,8}$ resulting from two-loop diagrams. In terms of the effective coefficients,

$$\begin{aligned}
C_7^{eff}(\mu) &= C_7(\mu) + \sum_I y_I C_I(\mu) \\
C_8^{eff}(\mu) &= C_8(\mu) + \sum_I z_I C_I(\mu),
\end{aligned} \tag{9}$$

with the index I running for 26 indices labeled by i and jq for the operators O_i ($i \neq 7, 8$) and O_j^q , the effective anomalous dimension matrix is given by

$$\gamma_{JI}^{eff} = \begin{cases} \gamma_{J7} + \sum_K y_K \gamma_{JK} - y_J \gamma_{77} - z_J \gamma_{87}, & \text{for } I = 7, J \neq 7, 8 \\ \gamma_{J8} + \sum_K z_K \gamma_{JK} - z_J \gamma_{88}, & \text{for } I = 8, J \neq 7, 8 \\ \gamma_{JI}, & \text{otherwise.} \end{cases} \tag{10}$$

The corresponding RG equations for the effective coefficients are then

$$\frac{d}{d \ln \mu} C_I^{eff}(\mu) = \frac{\alpha_s}{4\pi} \gamma_{JI}^{eff} C_J^{eff}(\mu). \tag{11}$$

In the naive dimensional regularization scheme which we follow [19], the nonvanishing coefficients y_I, z_I are

$$\begin{aligned}
y_5 &= y_{5b} = -\frac{1}{3}, & y_6 &= y_{6b} = -1 \\
z_5 &= z_{5b} = -1.
\end{aligned} \tag{12}$$

Now, keeping track of the flavor structure of the standard calculation of the matrix γ^{eff} [18], one can find the 28×28 anomalous dimension matrix in a straightforward way. The results are presented in the Appendix. There, we also calculate the evolution matrix from which the Wilson coefficients at the scale $\mu = m_b$ can be obtained in terms of the coefficients determined at the weak scale. The contributions to these coefficients from R-parity violation will be discussed in detail in the following sections.

III. CP ASYMMETRIES WITH LEPTON NUMBER VIOLATION

A. $\lambda'_{n3j} \lambda'^*_{n2j}$: Direct CP asymmetry

With the nonvanishing combination of the R-parity violating couplings $\lambda'_{n3j} \lambda'^*_{n2j}$, the effective Hamiltonian in Eq. (8) includes only the operators of the left-handed type: $O_{6L}^{d_j}$ induced by the tree-level diagram exchanging sneutrino $\tilde{\nu}_n$. In addition to these, there arises also additional effective semileptonic operators,

$$O_{9L}^{\nu_n} = \overline{s_L} \gamma^\mu b_L \overline{\nu_{nL}} \gamma_\mu \nu_{nL}, \tag{13}$$

through the tree diagram exchanging the right-handed down squark \tilde{d}_j^c . The corresponding Wilson coefficients are given by

$$\begin{aligned}
C_{6L}^{d_j}(m_W) &= \frac{-1}{4\sqrt{2}G_F\lambda_t} \sum_{n=1}^3 \frac{\lambda'_{n3j} \lambda'^*_{n2j}}{m_{\tilde{\nu}_n}^2}, \\
C_{9L}^{\nu_n}(m_W) &= \frac{1}{4\sqrt{2}G_F\lambda_t} \sum_{j=1}^3 \frac{\lambda'_{n3j} \lambda'^*_{n2j}}{m_{\tilde{d}_j^c}^2}.
\end{aligned} \tag{14}$$

Note that $C_{5L}^{d_j}(m_W) = 0$ and it remains vanishing at low energy scale under one-loop RG evolution as can be seen from Eq. (A9) in the Appendix. The R-parity violating contributions to the coefficients of the operators O_{7L} and O_{8L} come from one-loop diagrams exchanging sneutrinos and right-handed down squarks, and have been computed in Ref. [17]. They can be conveniently re-written in terms of the coefficients $C_{6L}^{d_j}$ and $C_{9L}^{\nu_n}$ as follows:

$$C_{7L}^{R_p}(m_W) = Q_d C_{8L}^{R_p}(m_W) = \frac{Q_d}{6} \left[\sum_n C_{9L}^{\nu_n}(m_W) + 2 \sum_j C_{6L}^{d_j}(m_W) \right], \quad (15)$$

where $Q_d = -1/3$. In deriving $C_{7L,8L}^{R_p}$, we neglected the down-type squark mixing and the masses of down-type quarks compared to the mass of the sneutrino. As shown in Eq. (12), the effective coefficient C_{7L}^{eff} at $\mu = m_W$ gets a nontrivial contribution from $C_{6L}^{d_j}$ with $j = 3$ and thus we have

$$\begin{aligned} C_{7L}^{eff}(m_W) &= C_{7L}(m_W) - C_{6L}^b(m_W), \\ C_{8L}^{eff}(m_W) &= C_{8L}(m_W). \end{aligned} \quad (16)$$

Making use of the relation (15) and the formula (A10) in the Appendix, one can obtain the Wilson coefficients at the scale $\mu = m_b$ as follows;

$$\begin{aligned} C_{7L}^{eff}(m_b) &= 0.67 C_{7L}^{SSM}(m_W) + 0.092 C_{8L}^{SSM}(m_W) - 0.17 C_{2L}(m_W) \\ &\quad - 0.14 [C_{6L}^d(m_W) + C_{6L}^s(m_W)] - 0.80 C_{6L}^b(m_W) - 0.022 C_{9L}^{\nu_n}(m_W), \\ C_{8L}^{eff}(m_b) &= 0.70 C_{8L}^{SSM}(m_W) - 0.080 C_{2L}(m_W) \\ &\quad + 0.42 [C_{6L}^d(m_W) + C_{6L}^s(m_W)] + 0.60 C_{6L}^b(m_W) + 0.12 C_{9L}^{\nu_n}(m_W), \end{aligned} \quad (17)$$

where $C_{7L,8L}^{SSM}(m_W)$ contain the contributions from the R-parity conserving MSSM sector as well as from the SM one. We assume that $C_{2L}(m_W)$ comes solely from the SM. To quantify $C_{7L,8L}^{SSM}(m_W)$ for our purpose, we introduce the parameters $\eta_{7,8}$ which are defined by

$$C_{7L}^{SSM}(m_W) \equiv \eta_7 C_{7L}^{SM}(m_W), \quad C_{8L}^{SSM}(m_W) \equiv \eta_8 C_{8L}^{SM}(m_W). \quad (18)$$

The parameters $\eta_{7,8}$ are complex in general.

Given the Wilson coefficients in Eq. (17), we are ready to analyze the CP violating effects from R-parity violation in the radiative B decay. Referring to the work by Kagan and Neubert [1] for details, the direct CP asymmetry A_{CP} and the branching ratios for the decays $b \rightarrow s\gamma$ and $b \rightarrow sg$ are given by ¹

$$\begin{aligned} A_{CP} &= \frac{1}{|C_{7L}|^2} \{ 1.23 \Im[C_{2L} C_{7L}^*] - 9.52 \Im[C_{8L} C_{7L}^*] + 0.10 \Im[C_{2L} C_{8L}^*] \} \quad (\%), \\ B(B \rightarrow X_s \gamma) &\approx 2.57 \times 10^{-3} K_{\text{NLO}}(\delta) \left(\frac{B_{\text{semi}}}{0.105} \right), \\ B(B \rightarrow X_{sg}) &\approx 0.96 |C_{8L}|^2 B_{\text{semi}}, \end{aligned} \quad (19)$$

¹Here, we neglect the R-parity violating contributions to A_{CP} through terms such as $\Im[C_L^{R_p} C_{7L}^*]$.

where the coefficients C 's without arguments are understood to be the effective ones evaluated at the scale m_b . The quantity $K_{\text{NLO}}(\delta) = |C_7|^2 + O(\alpha_s, 1/m_b^2)$ contains the corrections to the leading-order result and the specific forms of the corrections can be found in Ref. [1]. We will take $\delta = 0.3$ and $B_{\text{semi}} = 10.5\%$ for the branching ratio of the semileptonic decay $B \rightarrow X_c e \bar{\nu}$. In this work, we take the following values for the SM predictions for the $C_{2L,7L,8L}^{\text{SM}}$ at the m_W scale;

$$C_{2L}^{\text{SM}}(m_W) \approx 1.0, \quad C_{7L}^{\text{SM}}(m_W) \approx -0.20, \quad C_{8L}^{\text{SM}}(m_W) \approx -0.10. \quad (20)$$

The above choice of parameters yields the values at the scale $\mu = m_b = 4.8 \text{ GeV}$;

$$C_{2L}^{\text{SM}}(m_b) \approx 1.11, \quad C_{7L}^{\text{SM}}(m_b) \approx -0.32, \quad C_{8L}^{\text{SM}}(m_b) \approx -0.15. \quad (21)$$

Considering the above values of the SM coefficients, Eq. (17) suggests that a significant contribution from R-parity violation to the $b \rightarrow s\gamma$ decay can arise for $|C_{9L}^{\nu_n}(m_W)| \sim 10$, $|C_{6L}^{d,s}(m_W)| \sim 2$, or $|C_{6L}^b(m_W)| \sim 0.3$. Furthermore, as we will show, if a sizable $|C_{6L}^b| \sim 0.3$ is allowed, one can get the direct CP asymmetry of the order $|A_{CP}| \sim 10\%$ satisfying the observed branching ratios of $B \rightarrow X_s\gamma$ and $B \rightarrow X_{sg}$ decay.

In order to figure out how large CP asymmetry can come from R-parity violation, let us consider the experimental bounds on the new Wilson coefficients appeared in Eq. (17). First of all, those coefficients will be constrained by the experimental data for the branching ratios in Eq. (19). In this work, we use the CLEO data:

$$B(B \rightarrow X_s\gamma) = (3.15 \pm 0.35_{\text{stat}} \pm 0.32_{\text{stat}} \pm 0.26_{\text{model}}) \times 10^{-4}, \quad [20]$$

$$B(B \rightarrow X_{sg}) \lesssim 6.8\% \quad (90\% \text{C.L.}). \quad [21]$$

More important constraints on the coefficients C_{6L}^{dj} and $C_{9L}^{\nu_n}$ come from experimental data on the various B meson decays. Let us discuss the relevant bound for each coefficient.

First, the coefficient C_{6L}^d is constrained by the B decay mode $\overline{B}^0 \rightarrow K^0\pi^0$, whose branching ratio is observed to be [22]

$$B(\overline{B}^0 \rightarrow K^0\pi^0) < 4.1 \times 10^{-5}.$$

Following the similar method used in Ref. [23], we estimate the matrix element of the operator O_{6L}^d as

$$\langle K^0\pi^0 | O_{6L}^d | \overline{B}^0 \rangle \approx i \frac{m_K^2(m_B^2 - m_\pi^2)}{2(m_s + m_d)(m_b - m_d)} f_K F_1^{B \rightarrow \pi}(m_K^2). \quad (22)$$

Taking $f_K = 0.16 \text{ GeV}$, $F_1^{B \rightarrow \pi}(m_K^2) = 0.33$, we obtain

$$|C_{6L}^d| < 0.17. \quad (23)$$

Second, the coefficient C_{6L}^s is constrained by considering the B decay mode $\overline{B}^0 \rightarrow \phi K^0$. The experimental limits on the branching ratio of this decay mode is [24]

$$B(\overline{B}^0 \rightarrow \phi K^0) < 3.1 \times 10^{-5}.$$

From our estimation of the matrix element of the operator O_{6L}^s ;

$$\langle \phi K^0 | O_{6L}^s | \overline{B^0} \rangle \approx \frac{1}{4N} f_\phi m_\phi \epsilon \cdot (P_B + P_K) F_1^{B \rightarrow K}(m_\phi^2), \quad (24)$$

where ϵ is a polarization vector of ϕ and we will take $N = 3$. With $f_\phi = 0.23$ GeV, $F_1^{B \rightarrow K^0}(m_\phi^2) = 0.38$, we obtain following limit :

$$|C_{6L}^s| < 0.23. \quad (25)$$

The above bounds in Eqs. (23) and (25) tell us that the coefficients $C_{6L}^{d,s}$ cannot play any important role in the radiative B decays as can be seen from Eq. (17)².

Now let us consider the constraint on C_{6L}^b . The most useful bound comes *indirectly* from the consideration of the decay mode $\overline{B^0} \rightarrow X_s \nu_n \overline{\nu}_n$ whose branching ratio can be calculated as [25]

$$\text{B}(\overline{B^0} \rightarrow X_s \nu_n \overline{\nu}_n) = \left| \frac{\lambda_t}{V_{cb}} \right|^2 \frac{|C_{9L}^{\nu_n}|^2}{f_{\text{PS}}(m_c^2/m_b^2)} \text{B}_{\text{semi}}. \quad (26)$$

According to the analyses in Ref. [25], one obtains *indirect* experimental information on the above branching ratio:

$$\text{B}(\overline{B^0} \rightarrow X_s \nu_n \overline{\nu}_n) < 3.9 \times 10^{-4}.$$

Taking this value and $|\lambda_t/V_{cb}| = 0.976$, $f_{\text{PS}}(m_c^2/m_b^2) = 0.5$, we put the bound,

$$|C_{9L}^{\nu_n}| < 0.044. \quad (27)$$

Thus the contribution of the coefficient $C_{9L}^{\nu_n}$ to Eq. (17) can also be neglected. Now, under the condition that the bound (27) is applied to each component of the coefficient [see Eq. (14)], we get

$$|C_{6L}^b| < 0.044 \left(\frac{m_{d_3}^2}{m_{\nu_n}^2} \right), \quad (28)$$

for each $n = 1, 2, 3$. Here we remark that under the condition (27), the R-parity violating contribution to the semileptonic decay $B \rightarrow X_c l_n \overline{\nu}_n$ through the effective operator

$$\mathcal{H}_{eff} = -\frac{\lambda'_{n3j} \lambda_{n2j}^*}{2m_{d_j^c}^2} \overline{c}_L \gamma^\mu b_L \overline{e}_{nL} \gamma_\mu \nu_{nL}, \quad (29)$$

can be made small enough to satisfy the *direct* experimental bounds [26];

²The bounds in Eqs. (23) and (25) are at the scale m_b . Practically, the Wilson coefficients induced by the R-parity violating couplings at the scale m_b are nearly the same as those at the scale m_W [see Eq. (A9)].

$$\begin{aligned}
|\lambda'_{233}\lambda'^*_{223}| &< 1.1 \times 10^{-3} \left(\frac{m_{\tilde{d}_3^c}}{100 \text{ GeV}} \right)^2, \\
|\lambda'_{333}\lambda'^*_{323}| &< 4.4 \times 10^{-3} \left(\frac{m_{\tilde{d}_3^c}}{100 \text{ GeV}} \right)^2.
\end{aligned} \tag{30}$$

Finally, we have to consider the neutrino mass coming from our choice of nonvanishing $\lambda'_{n33}\lambda'^*_{n23}$. With nonzero value of λ'_{n33} , the neutrino ν_n may get an undesirably large mass from one-loop diagrams with the exchange of bottom quark and squark [27]. The one-loop contribution to the neutrino mass is given by

$$m_{\nu_n} \approx \frac{3}{8\pi^2} \frac{\lambda'^2_{n33} A m_b^2}{\tilde{m}^2}$$

where A denotes left-right sbottom mixing parameter and \tilde{m} is an average sbottom mass. Taking $\lambda'^2_{n33}/\tilde{m}^2 < 10^{-7} \text{ GeV}^{-2}$ from the consideration of Eq. (27), we obtain

$$m_{\nu_n} \lesssim 10 \text{ keV} \tag{31}$$

with $A = 100 \text{ GeV}$. Thus it is well below the direct experimental limit for the muon (tau) neutrino which is 0.17 (18) MeV [22]. Yet another indirect limit for the muon or tau neutrino mass comes from the observation of atmospheric muon neutrinos. The atmospheric neutrino data from the Super-Kamiokande are known to be nicely explained by the neutrino oscillation between largely mixed muon and tau neutrinos [28]. A natural consequence of this would be that the muon and tau neutrinos are very light $m_{\nu_\mu, \nu_\tau} \lesssim 1 \text{ eV}$. If this is the case, there is no room at all for large CP asymmetry from R-parity violation. However, having a large R-parity violation and thus a heavier muon or tau neutrino is not excluded completely as there exist some other viable options for the explanation of the Super-Kamiokande data, such as a neutrino decay [29].

From the above consideration, the only possibility for a significant enhancement of R-parity violating contribution to the $b \rightarrow s\gamma$ decay is to have a large coefficient C_{6L}^b with a sfermion mass hierarchy $m_{\tilde{d}_3^c} > m_{\tilde{\nu}_n}$. For example, we need $m_{\tilde{d}_3^c} \sim 5 m_{\tilde{\nu}_n}$ to get $C_{6L}^b \sim 1$. Taking into account all the above experimental limits on the Wilson coefficients, let us now analyze how large CP asymmetry A_{CP} can be obtained. In FIG. 2, we show A_{CP} as a function of the branching ratio of the decay $B \rightarrow X_s\gamma$ varying $|C_{6L}^b|$ and $\text{Arg}(C_{6L}^b)$ from 0 to 2π . The other R-parity violating couplings are neglected. We take $\eta_7 = \eta_8 = \eta$ as a real number in FIG. 2 even though η is a complex number generally. In this figure we consider additional experimental constraints coming from $B(B \rightarrow X_s\gamma)$, $B(B \rightarrow X_{sg})$. If $m_{\tilde{d}_3^c}^2$ are larger than $m_{\tilde{\nu}_n}^2$, then sizable $|C_{6L}^b|$ is allowed evading the bounds Eq. (28). We find the CP asymmetry can reach 13 % for $m_{\tilde{d}_3^c}/m_{\tilde{\nu}_n} \approx 3.4$ with vanishing R-parity conserving supersymmetric contributions ($\eta = 1$) as shown in the left-upper frame of FIG. 2. In the case where the R-parity conserving supersymmetric contributions take the same sign as the SM values of $C_{7L,8L}(m_W)$, this CP asymmetry can be larger as shown in FIG. 2 with $\eta = 2$. On the other hand, the CP asymmetry decreases when the R-parity conserving supersymmetric contributions take the opposite sign to the SM values of $C_{7L,8L}(m_W)$ as seen from FIG. 2 with $\eta = 0, -1$.

Before concluding this subsection, it is worthwhile to notice that the CP asymmetry in the hadronic B decays such as $B_d \rightarrow \pi K_S$ and $B_d \rightarrow \phi K_S$ can be significantly affected by

the R-parity violating couplings $C_{6L}^{d,s}$ even if the effects of the couplings on the direct CP asymmetry in the radiative B decay are negligible [13–15].

B. $\lambda'_{nj2}\lambda'^*_{nj3}$: Mixing-induced CP asymmetry

Contrary to the previous case, the combination of R-parity violating couplings $\lambda'_{nj2}\lambda'^*_{nj3}$ leads only to the right-handed set of operators: $O_{7R,8R}$ and $O_{5R,6R}^q$ in the effective Hamiltonian (8). In addition, the coefficient of the operator O_{6R}^t is also generated. Even though it does not contribute to the b decay, it's coefficient will be considered since it enters into $C_{7R,8R}$. As in the previous subsection, we have also the semileptonic four-Fermi operators as follows:

$$\begin{aligned} O_{9R}^{\nu_n} &= \overline{s_R^\alpha} \gamma^\mu b_R^\alpha \overline{\nu_{nL}} \gamma_\mu \nu_{nL}, \\ O_{10R}^{e_n} &= \overline{s_R^\alpha} \gamma^\mu b_R^\alpha \overline{e_{nL}} \gamma_\mu e_{nL}. \end{aligned} \quad (32)$$

The nonzero Wilson coefficients at m_W induced from our R-parity violation are

$$\begin{aligned} C_{6R}^{qj}(m_W) &= \frac{-1}{4\sqrt{2}G_F\lambda_t} \sum_{n=1}^3 \frac{\lambda'_{nj2}\lambda'^*_{nj3}}{m_{\tilde{l}_n}^2}, \\ C_{9R}^{\nu_n}(m_W) &= C_{10R}^{e_n}(m_W) = \frac{1}{4\sqrt{2}G_F\lambda_t} \sum_{j=1}^3 \frac{\lambda'_{nj2}\lambda'^*_{nj3}}{m_{\tilde{Q}_j}^2}, \end{aligned} \quad (33)$$

where q_j can be either u_j or d_j , $m_{\tilde{l}_n}$ and $m_{\tilde{Q}_j}$ are the masses of the doublet slepton and squark, respectively. In deriving these coefficients, we do not consider the effect of the CKM mixing. Note that there can be also other four-Fermi operators involving two different flavors of quarks or leptons through the CKM mixing, which we neglect in our discussion as they give subleading contributions. Now, disregarding the contributions from the R-parity conserving supersymmetric sector, the Wilson coefficients for $O_{7R,8R}$ at the scale m_W [17] can be expressed in terms of the coefficients in Eq. (33) as follows

$$\begin{aligned} C_{7R}(m_W) &= C_{7R}^{R_p}(m_W) = \frac{Q_d}{6} \left[\sum_n C_{9R}^{\nu_n}(m_W) + 2 \sum_j C_{6R}^{qj}(m_W) \right] \\ &\quad + \frac{1}{18} \left[8 \sum_n C_{10R}^{e_n}(m_W) + 7 \sum_j C_{6R}^{qj}(m_W) P_\gamma(x_j) \right], \end{aligned} \quad (34)$$

$$\begin{aligned} C_{8R}(m_W) &= C_{8R}^{R_p}(m_W) = \frac{1}{6} \left[\sum_n C_{9R}^{\nu_n}(m_W) + 2 \sum_j C_{6R}^{qj}(m_W) \right. \\ &\quad \left. + \sum_n C_{10R}^{e_n}(m_W) + 2 \sum_j C_{6R}^{qj}(m_W) P_g(x_j) \right], \end{aligned} \quad (35)$$

where

$$P_\gamma(x_j) = \frac{36}{7} [Q_u F_1(x_j) + F_2(x_j)], \quad P_g(x_j) = 6F_1(x_j)$$

with $x_j = m_{u_j}^2/m_{\tilde{l}_n}^2$. The functions $F_{1,2}$ are defined as

$$F_1(x) = \frac{1}{12} \frac{(2 + 3x - 6x^2 + x^3 + 6x \ln x)}{(1-x)^4},$$

$$F_2(x) = \frac{1}{12} \frac{(1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x)}{(1-x)^4}.$$

Note that these functions are defined as $P_\gamma(0) = P_g(0) = 1$ and their values for some selected x are listed in TABLE II

In the case under consideration, there is no phase in $C_{7R}C_{8R}^*$ contributing the direct CP asymmetry in right-handed sector analogue of Eq. (19) since the same combination of the couplings generates both $C_{7R,8R}$. However, there can arise a sizable mixing-induced CP asymmetry as defined in Eq. (2). That is, we may have $|A_M| \approx 1$ with $|C_{7L}| \approx |C_{7R}|$. Here, the Wilson coefficients are effective ones evaluated at the scale m_b . Combining our results in the Appendix (A11) and Eq. (34), we find

$$C_{7R}^{eff}(m_b) = 0.40 [C_{6R}^{q_1}(m_W) + C_{6R}^{q_2}(m_W)] + [-0.80 + 0.26 P_\gamma(x_t) + 0.031 P_g(x_t)] C_{6R}^{q_3}(m_W) - 0.022 C_{9R}^{\nu_n}(m_W) + 0.31 C_{10R}^e(m_W). \quad (36)$$

From this equation, it appears that we can easily obtain $C_{7R} \approx |C_{7L}| = |C_{7L}^{SSM}| \approx 0.32 |\eta_7|$. To clarify this, we now consider the experimental constraints on the coefficients appearing in Eq. (36). Let us first note that the arguments in the previous subsection are applied here to get the bounds;

$$|C_{6R}^{q_1}| < 0.17, \quad |C_{6R}^{q_2}| < 0.23, \quad |C_{9R}^{\nu_n}| < 0.044 \quad (37)$$

as in Eqs. (23), (25) and (27), respectively. Thus, the contributions of these coefficients in Eq. (36) are not significant. Another important constraint on the relevant R-parity violating couplings comes from the decay $B \rightarrow X_s l_n^+ l_n^-$ [30] induced by the second effective operator in Eq. (32). This consideration leads to

$$|C_{10R}^e| < 0.017 \left(\sqrt{\frac{B(b \rightarrow X_s l_n^+ l_n^-)^{\text{expt}}}{5.7 \times 10^{-5}}} \right), \quad (38)$$

where $B(b \rightarrow X_s l_n^+ l_n^-)^{\text{expt}}$ denotes the experimental upper limit on the branching ratio of the $B \rightarrow X_s l_n^+ l_n^-$ decay modes given by [22]

$$B(B \rightarrow X_s e^+ e^-) < 5.7 \times 10^{-5},$$

$$B(B \rightarrow X_s \mu^+ \mu^-) < 5.8 \times 10^{-5}.$$

Even if there are no data for $b \rightarrow X_s \tau^+ \tau^-$, we expect it's branching ratio is most probably less than that of $B \rightarrow X_s e^+ e^-$. Then, similarly to Eq. (28), the bounds on C_{6R}^q can be obtained indirectly from Eq. (38) as

$$|C_{6R}^{q_j}| < 0.017 \left(\sqrt{\frac{B(B \rightarrow X_s l_n^+ l_n^-)^{\text{expt}}}{5.7 \times 10^{-5}}} \right) \left(\frac{m_{\tilde{Q}_j}}{m_{\tilde{l}_n}} \right)^2, \quad (39)$$

for each j and n . Therefore, we may obtain a sizable $C_{7R}^{eff}(m_b)$ from the contribution of $C_{6R}^{q3}(m_W)$ if there is again a hierarchy between sfermion masses. For example, taking $m_{\tilde{Q}_3}/m_{\tilde{t}_n} \approx 5$, $|C_{6R}^{q3}| = 0.33$ is allowed within the present experimental bound. Taking $x_t = 1$ which gives $P_\gamma(1) = 5/14$ and $P_g(1) = 1/4$, one could obtain $|C_{7R}^{eff}(m_b)| \approx 0.23$. If $|C_{7L}^{eff}(m_b)| = 0.23$, $|A_M| \approx 1$ is possible accommodating the measured $B(b \rightarrow s\gamma)$. Finally, let us note that a large mixing-induced CP asymmetry requires the (tau) neutrino to be heavy as discussed in the previous subsection.

Again we note that with the coupling $\lambda'_{n22}\lambda_{n32}^*$ which does not affect the radiative B decays, one can have important effects on the CP asymmetries in the hadronic B decays such as $B_d \rightarrow \phi K_S$ [13,14] and $B^\pm \rightarrow \pi^\pm K^0$ [15].

IV. CP ASYMMETRIES WITH BARYON NUMBER VIOLATION

$\lambda''_{n12}\lambda_{n13}^*$: Mixing-induced CP asymmetry

Our final case is to have baryon number violation while lepton number is conserved. Then, the new operator set for the $b \rightarrow s$ transition contains again only right-handed ones with nonvanishing product of couplings $\lambda''_{n12}\lambda_{n13}^*$: $O_{3R,4R}^{u_n,d}$. The Wilson coefficients of these operators calculated at the weak scale m_W are

$$\begin{aligned} C_{3R}^{u_n}(m_W) &= \frac{1}{4\sqrt{2}G_F\lambda_t} \frac{\lambda''_{n12}\lambda_{n13}^*}{m_{\tilde{d}_1^c}^2} = -C_{4R}^{u_n}, \\ C_{3R}^d(m_W) &= \frac{1}{4\sqrt{2}G_F\lambda_t} \sum_{n=1}^3 \frac{\lambda''_{n12}\lambda_{n13}^*}{m_{\tilde{u}_n^c}^2} = -C_{4R}^d. \end{aligned} \quad (40)$$

Notice that the simultaneous presence of nonvanishing coefficients $C_{3R,4R}$ is due to color antisymmetry in the superpotential term, $U^c D^c D^c$. Following the similar steps as before, we get the relation

$$\begin{aligned} C_{7R}(m_W) &= C_{7R}^{R_p}(m_W) = \frac{1}{9} \left[4C_{4R}^d(m_W) - 5 \sum_n C_{4R}^{u_n}(m_W) P'_\gamma(x_n) \right], \\ C_{8R}(m_W) &= C_{8R}^{R_p}(m_W) = \frac{1}{6} \left[C_{4R}^d(m_W) + \sum_n C_{4R}^{u_n}(m_W) P'_g(x_n) \right], \end{aligned} \quad (41)$$

where

$$P'_\gamma(x_n) = \frac{36}{5} [Q_u F_1(x_n) - Q_d F_2(x_n)], \quad P'_g(x_n) = 12 [F_1(x_n) - F_2(x_n)]$$

with $x_n = m_{u_n}^2/m_{\tilde{d}^c}^2$ and $P'_\gamma(0) = P'_g(0) = 1$. The Wilson coefficient C_{7R}^{eff} for the $b \rightarrow s\gamma$ decay at m_b is

$$\begin{aligned} C_{7R}^{eff}(m_b) &= 0.21 C_{4R}^d(m_W) - 0.17 [C_{4R}^u(m_W) + C_{4R}^c(m_W)] \\ &\quad - [0.37 P'_\gamma(x_t) - 0.015 P'_g(x_t)] C_{4R}^t(m_W). \end{aligned} \quad (42)$$

Let us now consider the experimental limits for the various coefficients in Eq. (42). First of all, the R-parity violating couplings with $n = 1$ are strongly constrained by the non-observation of nucleon-antinucleon oscillation and double nucleon decay; $\lambda''_{113} < 5 \times 10^{-3}$ and $\lambda''_{112} < 10^{-6}$ with sfermion mass of 300 GeV [31]. Thus C_{4R}^u are negligibly small. The constraint for $C_{4R}^d(m_W)$ comes again from the $B \rightarrow K^0 \pi^0$ decay. We estimate the matrix element of the operator O_{4R}^d as

$$\langle K^0 \pi^0 | O_{4R}^d | \bar{B}^0 \rangle \approx i (m_B^2 - m_\pi^2) f_K F_1^{B \rightarrow \pi}(m_K^2). \quad (43)$$

Using the similar values used in Eq. (23), we obtain

$$|C_{4R}^d| < 0.15. \quad (44)$$

Note that this bound is also consistent with the data for the decay mode $B^+ \rightarrow \bar{K}^0 \pi^+$ [32]. Concerning the coefficient C_{4R}^c , the consideration of the $B \rightarrow J/\psi K_S$ decay gives [14]

$$|C_{4R}^c| < 0.02. \quad (45)$$

Applying again the bound (44) to each component of C_{4R}^d , we get the bound on the C_{4R}^t ;

$$|C_{4R}^t| < 0.15 \frac{m_{\tilde{t}c}^2}{m_{\tilde{d}c}^2}. \quad (46)$$

Thus, to get $|C_{7R}^{eff}| > 0.2$ leading to a nearly maximal mixing-induced CP asymmetry, we need a sizable C_{4R}^t which can come about for $m_{\tilde{t}c} > 3.5 m_{\tilde{d}c}$ with $x_t = 1$ ³.

We conclude this section by pointing out that the direct CP asymmetry in the decay $B^\pm \rightarrow \pi^\pm K^0$ can arise maximally as is the case with the coupling $\lambda'_{n22} \lambda_{n32}^*$ [15].

V. CONCLUSION

We have discussed the effects of R-parity violation on the CP asymmetries in radiative B decays. When we allow R-parity and lepton number violating couplings which generate at one-loop level the tau neutrino mass of order 10 keV, they can induce rather large CP violating effects in the $b \rightarrow s \gamma$ decay. The direct CP asymmetry can be as large as 17 % if the R-parity conserving supersymmetric contribution is comparable to the SM one. The mixing-induced CP asymmetry can be almost maximal depending on the sfermion masses. For these sizable CP violating effects, it is required to have moderate sfermion mass splittings by factor 4 or bigger. If the atmospheric neutrino data from the Super-Kamiokande are to be explained by the oscillation between the muon and tau neutrinos whose masses are very light $m_{\nu_\mu, \nu_\tau} \lesssim 1$ eV, then the effects of R-parity violation on the radiative B decays are

³The constraint on the relevant single baryon number violating coupling comes from $\Gamma(Z \rightarrow l\bar{l})/\Gamma(Z \rightarrow \text{hadrons})$ [33], which gives $|\lambda''_{312,313,323}| < 0.5$ for $\tilde{m} = 100$ GeV. This constraint is so weak that $|C_{3R,4R}^t| \sim \mathcal{O}(1)$ is easily allowed.

negligible. A large mixing-induced CP asymmetry is also possible with the R-parity and baryon number violating couplings with the similar order of sfermion mass splitting.

These results could be contrasted with the R-parity violating effects on the CP asymmetries in the hadronic B decays such as $B_d \rightarrow J/\psi K, \phi K$ or $B^\pm \rightarrow \pi^\pm K$, which could be significant without sfermion mass splitting and are rather insensitive to the neutrino mass restriction.

Note added: While our work was being prepared, we encountered the paper [34] which considers the R-parity violating effect on the radiative B decay. It also deals with the RG running of the enlarged set of the Wilson coefficients which overlaps partly with our paper, and we find discrepancies in anomalous dimension matrix elements and the relation like (A10) and (A11).

ACKNOWLEDGMENTS

We thank G. Bhattacharyya for helpful discussions. The work of KH was supported by grant No. 1999-2-111-002-5 from the interdisciplinary Research program of the KOSEF and BK21 project of the Ministry of Education.

APPENDIX A: RG EQUATIONS WITH THE EXTENDED OPERATORS

Here, we present the anomalous dimension matrix for the 28 operators including the standard set $O_{1,\dots,8}$ and the extended set $O_{3,\dots,6}^q$ with $q = u, d, s, c, b$. We drop the indices L, R for the left-handed and right-handed set of operators as they have the identical anomalous dimension matrix. Omitting the usual 8×8 matrix for the standard set of operators, we have the following nonvanishing block-diagonal elements of the whole 28×28 matrix.

The 2×2 submatrices mixing the operators $O_{3,4}^q$ and $O_{5,6}^q$ with themselves are

$$\begin{pmatrix} -2 & 6 \\ 6 & -2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 & -6 \\ 0 & -16 \end{pmatrix}, \quad (\text{A1})$$

respectively. The 2×8 submatrix mixing the operators $O_{3,4}^q$ with the standard set $O_{1,\dots,8}$ is given by

$$\begin{pmatrix} 0 & 0 & -\frac{2}{9}\delta_{qb,s} & \frac{2}{3}\delta_{qb,s} & -\frac{2}{9}\delta_{qb,s} & \frac{2}{3}\delta_{qb,s} & -\frac{232}{81}\delta_{qb,s} & 3 + \frac{70}{27}\delta_{qb,s} \\ 0 & 0 & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & \frac{416}{81}\delta_{qu} - \frac{232}{81}\delta_{qd} & \frac{70}{27} + 3\delta_{qb,s} \end{pmatrix} \quad (\text{A2})$$

where

$$\delta_{qb,s} = \begin{cases} 1 & \text{for } q = b \text{ or } s \\ 0 & \text{otherwise,} \end{cases}$$

$$\delta_{qu}, \delta_{qd} = \begin{cases} 1 & \text{when } q \text{ is an up-type, or down-type quark} \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the 2×8 submatrix mixing the operators $O_{5,6}^q$ with the 8 standard operators is

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -\frac{32}{9}\delta_{qb} & -3 - \frac{14}{3}\delta_{qb} \\ 0 & 0 & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & -\frac{448}{81}\delta_{qu} + \frac{200}{81}\delta_{qd} & -\frac{119}{27} - 4\delta_{qb} \end{pmatrix} \quad (\text{A3})$$

where

$$\delta_{qb} = \begin{cases} 1 & \text{for } q = b \\ 0 & \text{otherwise.} \end{cases}$$

With this anomalous dimension matrix, the low energy Wilson coefficients are given by

$$\vec{C}^{eff}(\mu) = \hat{U}^{eff}(\mu, \mu_W) \vec{C}^{eff}(\mu_W) \quad (\text{A4})$$

where

$$\hat{U}^{eff}(\mu, \mu_W) = \hat{V} \left(\left[\frac{\alpha_s(\mu_W)}{\alpha_s(\mu)} \right]^{\frac{\vec{\gamma}^{eff}}{2\beta_0}} \right)_D \hat{V}^{-1} \quad (\text{A5})$$

where \hat{V} diagonalizes $\hat{\gamma}^{effT}$

$$\hat{\gamma}_D^{eff} = \hat{V}^{-1} \hat{\gamma}^{effT} \hat{V} \quad (\text{A6})$$

and $\vec{\gamma}^{eff}$ is the vector containing the diagonal elements of the diagonal matrix of eigenvalues of $\hat{\gamma}^{eff}$.

By solving Eqs. (A4)–(A6), we find the following low energy Wilson coefficients at μ_b in terms of nonzero $C_6^{d_i}(\mu_W)$, $C_6^{u,c}(\mu_W)$ in the L or R sector induced at μ_W by R parity violating coupling $\lambda'_{n2j}\lambda_{n3j}^*$ or $\lambda'_{nj2}\lambda_{nj3}^*$, respectively, and $C_{3,4}^{u,d,c}(\mu_W)$ in the R sector by $\lambda''_{n12}\lambda_{n13}''^*$, and the SM contribution $C_{2L}(\mu_W)$ in L sector:

$$\begin{aligned} C_7^{eff}(\mu_b) = & \eta^{16/23} C_7^{eff}(\mu_W) + \frac{8}{3} (\eta^{14/23} - \eta^{16/23}) C_8^{eff}(\mu_W) + C_2(\mu_W) \sum_{i=1}^{10} h_i \eta^{a_i} \\ & + (C_6^d(\mu_W) + C_6^s(\mu_W)) \sum_{i=1}^{10} r_i^1 \eta^{a_i} + C_6^b(\mu_W) \sum_{i=1}^{10} r_i^2 \eta^{a_i} + (C_6^u(\mu_W) + C_6^c(\mu_W)) \sum_{i=1}^{10} r_i^3 \eta^{a_i} \\ & + C_3^d(\mu_W) \sum_{i=1}^{10} r_i^4 \eta^{a_i} + C_4^d(\mu_W) \sum_{i=1}^{10} r_i^5 \eta^{a_i} \\ & + (C_3^u(\mu_W) + C_3^c(\mu_W)) \sum_{i=1}^{10} r_i^6 \eta^{a_i} + (C_4^u(\mu_W) + C_4^c(\mu_W)) \sum_{i=1}^{10} r_i^7 \eta^{a_i} \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} C_8^{eff}(\mu_b) = & \eta^{14/23} C_8^{eff}(\mu_W) + C_2(\mu_W) \sum_{i=1}^{10} \bar{h}_i \eta^{a_i} \\ & + (C_6^d(\mu_W) + C_6^s(\mu_W)) \sum_{i=1}^{10} \bar{r}_i^1 \eta^{a_i} + C_6^b(\mu_W) \sum_{i=1}^{10} \bar{r}_i^2 \eta^{a_i} + (C_6^u(\mu_W) + C_6^c(\mu_W)) \sum_{i=1}^{10} \bar{r}_i^3 \eta^{a_i} \\ & + C_3^d(\mu_W) \sum_{i=1}^{10} \bar{r}_i^4 \eta^{a_i} + C_4^d(\mu_W) \sum_{i=1}^{10} \bar{r}_i^5 \eta^{a_i} \\ & + (C_3^u(\mu_W) + C_3^c(\mu_W)) \sum_{i=1}^{10} \bar{r}_i^6 \eta^{a_i} + (C_4^u(\mu_W) + C_4^c(\mu_W)) \sum_{i=1}^{10} \bar{r}_i^7 \eta^{a_i} \end{aligned} \quad (\text{A8})$$

$$\begin{aligned}
C_2(\mu_b) &= \frac{1}{2} \left(\eta^{6/23} + \eta^{-12/23} \right) C_2(\mu_W) \\
C_5^{d,u,c}(\mu_b) &= 0 \\
C_6^{d,u,c}(\mu_b) &= C_6^{d,u,c}(\mu_W) \\
C_3^{u,d,c}(\mu_b) &= \frac{1}{2} \left(\eta^{6/23} + \eta^{-12/23} \right) C_3^{u,d,c}(\mu_W) + \frac{1}{2} \left(\eta^{6/23} - \eta^{-12/23} \right) C_4^{u,d,c}(\mu_W) \\
C_4^{u,d,c}(\mu_b) &= \frac{1}{2} \left(\eta^{6/23} + \eta^{-12/23} \right) C_4^{u,d,c}(\mu_W) + \frac{1}{2} \left(\eta^{6/23} - \eta^{-12/23} \right) C_3^{u,d,c}(\mu_W)
\end{aligned} \tag{A9}$$

where the quantities $a_i, h_i, \bar{h}_i, r_i^n, \bar{r}_i^n$ are shown in the TABLE I.

The explicit numerical expressions with the choice $\mu_W = m_W$, $\mu_b = 4.8$ GeV, $\alpha_s(\mu_W) = 0.120$ and $\alpha_s(\mu_b) = 0.214$ are

$$\begin{aligned}
C_{7L}^{eff}(\mu_b) &= 0.6687 C_{7L}(\mu_W) + 0.0920 C_{8L}(\mu_W) - 0.1732 C_{2L}(\mu_W) \\
&\quad - 0.0974 \left(C_{6L}^d(\mu_W) + C_{6L}^s(\mu_W) \right) - (0.6687 + 0.0875) C_{6L}^b(\mu_W) \\
C_{8L}^{eff}(\mu_b) &= 0.7032 C_{8L}(\mu_W) - 0.0801 C_{2L}(\mu_W) \\
&\quad + 0.1893 \left(C_{6L}^d(\mu_W) + C_{6L}^s(\mu_W) \right) + 0.3670 C_{6L}^b(\mu_W)
\end{aligned} \tag{A10}$$

$$\begin{aligned}
C_{7R}^{eff}(\mu_b) &= 0.6687 C_{7R}(\mu_W) + 0.0920 C_{8R}(\mu_W) \\
&\quad - 0.0974 \left(C_{6R}^d(\mu_W) + C_{6R}^s(\mu_W) \right) - (0.6687 + 0.0875) C_{6R}^b(\mu_W) \\
&\quad + 0.2506 \left(C_{6R}^u(\mu_W) + C_{6R}^c(\mu_W) \right) - 0.0170 C_{3R}^d(\mu_W) + 0.0880 C_{4R}^d(\mu_W) \\
&\quad + 0.0147 \left(C_{3R}^u(\mu_W) + C_{3R}^c(\mu_W) \right) - 0.1732 \left(C_{4R}^u(\mu_W) + C_{4R}^c(\mu_W) \right) \\
C_{8R}^{eff}(\mu_b) &= 0.7032 C_{8R}(\mu_W) \\
&\quad + 0.1893 \left(C_{6R}^d(\mu_W) + C_{6R}^s(\mu_W) \right) + 0.3670 C_{6R}^b(\mu_W) \\
&\quad + 0.1893 \left(C_{6R}^u(\mu_W) + C_{6R}^c(\mu_W) \right) \\
&\quad - 0.0894 \left(C_{3R}^d(\mu_W) + C_{3R}^u(\mu_W) + C_{3R}^c(\mu_W) \right) \\
&\quad - 0.0801 \left(C_{4R}^d(\mu_W) + C_{4R}^u(\mu_W) + C_{4R}^c(\mu_W) \right)
\end{aligned} \tag{A11}$$

where we have used $C_7^{eff}(\mu_W) = C_7(\mu_W) - C_6^b(\mu_W)$ and $C_8^{eff}(\mu_W) = C_8(\mu_W)$ on the right-hand sides of the above equations.

TABLES

i	1	2	3	4	5	6	7	8	9	10
a_i	$\frac{14}{23}$	$\frac{16}{23}$	$\frac{6}{23}$	$-\frac{12}{23}$	$-\frac{24}{13}$	$\frac{3}{23}$	0.4086	-0.4230	-0.8994	0.1456
h_i	2.2996	-1.0880	$-\frac{3}{7}$	$-\frac{1}{14}$	0	0	-0.6494	-0.0380	-0.0185	-0.0057
\bar{h}_i	0.8623	0	0	0	0	0	-0.9135	0.0873	-0.0571	0.0209
r_i^1	-0.1636	0.3413	0	0	-0.1242	0	-0.1140	-0.0141	0.0734	0.0013
r_i^2	-0.5847	0.7413	0	0	-0.1032	0	-0.1140	-0.0141	0.0734	0.0013
r_i^3	-0.1636	0.0413	0	0	0.1758	0	-0.1140	-0.0141	0.0734	0.0013
r_i^4	1.9233	-1.5327	0.1714	-0.1429	0	0	-0.4714	0.0508	0.0094	-0.0081
r_i^5	2.2996	-1.9023	0.1714	0.1429	0	0	-0.6494	-0.0380	-0.0185	-0.0057
r_i^6	1.9233	-1.1469	-0.4286	0.0714	0	0	-0.4714	0.0508	0.0094	-0.0081
r_i^7	2.2996	-1.0880	-0.4286	-0.0714	0	0	-0.6494	-0.0380	-0.0185	-0.0057
\bar{r}_i^1	-0.0613	0	0	-0.0316	0	0	-0.1604	-0.0325	0.2258	-0.0049
\bar{r}_i^2	-0.2192	0	0	0.1263	0	0	-0.1604	-0.0325	0.2258	-0.0049
\bar{r}_i^3	-0.0613	0	0	-0.0316	0	0	-0.1604	-0.0325	0.2258	-0.0049
\bar{r}_i^4	0.7212	0	0	0	0	0	-0.6631	-0.1168	0.0290	0.0296
\bar{r}_i^5	0.8623	0	0	0	0	0	-0.9135	0.0873	-0.0571	0.0209
\bar{r}_i^6	0.7212	0	0	0	0	0	-0.6631	-0.1168	0.0290	0.0296
\bar{r}_i^7	0.8623	0	0	0	0	0	-0.9135	0.0873	-0.0571	0.0209

TABLE I. The magic numbers with R-parity violation

x	$P_\gamma(x)$	$P_g(x)$	$P'_\gamma(x)$	$P'_g(x)$
$\left(\frac{170}{100}\right)^2$	0.197	0.121	0.157	-0.0558
1	5/14	1/4	3/10	0
$\left(\frac{170}{200}\right)^2$	0.414	0.301	0.354	0.0378
$\left(\frac{170}{300}\right)^2$	0.560	0.445	0.499	0.176
0	1	1	1	1

TABLE II. Values for the loop functions defined in the main text for several x 's.

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FIGURES

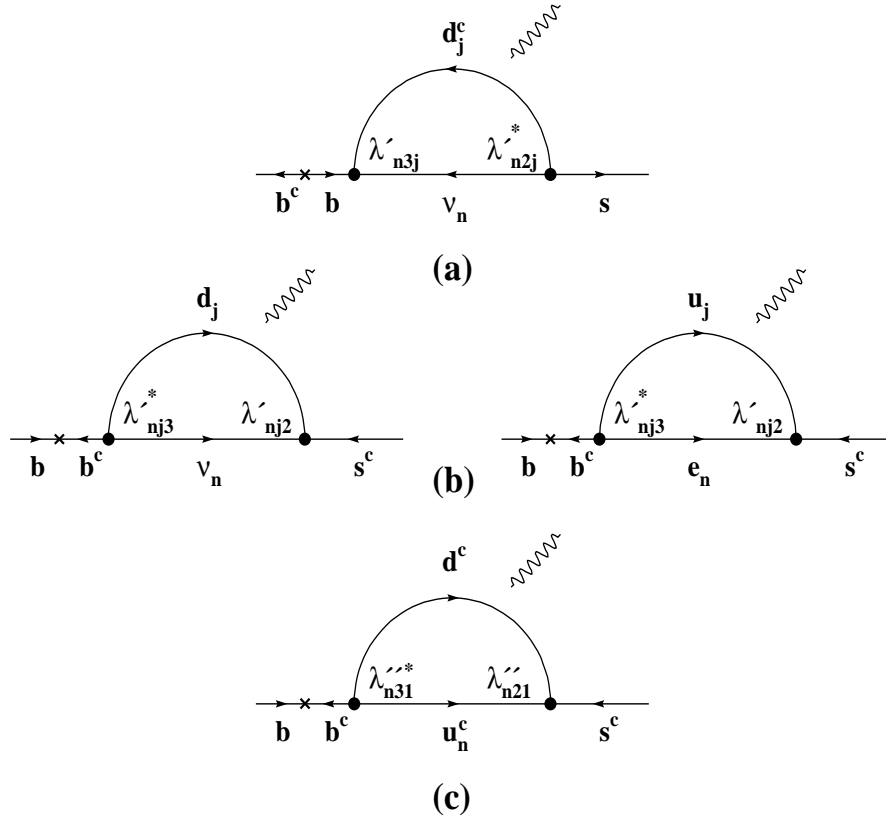


FIG. 1. One-loop diagrams generating dipole moment interactions for the $b \rightarrow s\gamma$ decay: the left-handed (a) and right-handed type (b) from the lepton number violating couplings, and the right-handed type (c) from the baryon number violating couplings.

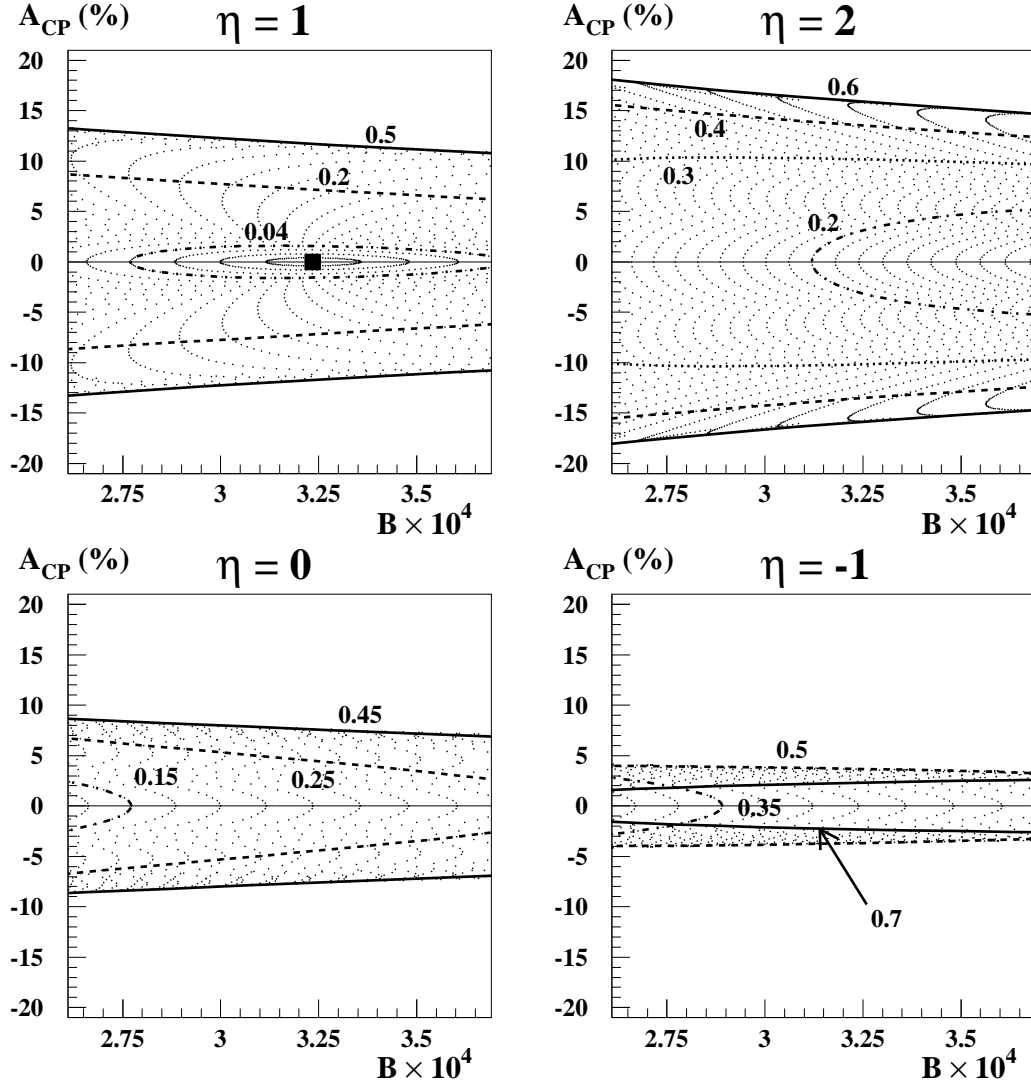


FIG. 2. Scattered plots for A_{CP} as a function of $B(B \rightarrow X_s \gamma) \times 10^4$ for the corresponding values of η . The SM prediction is marked as a filled square. The numbers are values of $|C_{6L}^b|$ for the corresponding contour lines. The values of parameters of each case are shown in the text.